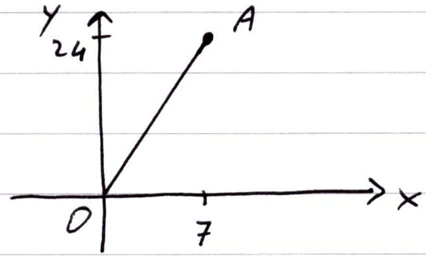
Understanding Mechanics, Sadler and Thorning

Exercise 2B : odd Numbered Ex only

(1)



$\vec{OA} = 7\underline{i} + 24\underline{j}$

$|\vec{OA}| = \sqrt{49 + 576} = 25\ m$

(3) given $\vec{OA} = 3\underline{i} - 4\underline{j} + 5\underline{k}$

$\vec{OB} = 8\underline{i} + 8\underline{j} - 3\underline{k}$

& $\vec{OC} = 4\underline{i} + 3\underline{k}$

Then (a) $|\vec{OA}| = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}\ m$

(b) $|\vec{OB}| = \sqrt{64 + 64 + 9} = \sqrt{137}\ m$

(c) $|\vec{OC}| = \sqrt{16 + 9} = 5\ m$

(d) $\vec{AB} = -\vec{OA} + \vec{OB} = 5\underline{i} + 12\underline{j} - 8\underline{k}$

(e) $\vec{BC} = -\vec{OB} + \vec{OC} = -4\underline{i} - 8\underline{j} + 6\underline{k}$

(f) $\vec{CB} = -\vec{BC} = 4\underline{i} + 8\underline{j} - 6\underline{k}$

(g) $|\vec{AB}| = \sqrt{25 + 144 + 64} = \sqrt{233}\ m$

(h) $|\vec{BC}| = \sqrt{16 + 64 + 36} = \sqrt{116}\ m = 2\sqrt{29}\ m$

(5) given $\underline{v} = 7\underline{i} - 24\underline{j}$ m/s

we have $|\underline{v}| = \sqrt{49+576} = \sqrt{625}$ m/s $= 25$ m/s

(7) given $\underline{v} = 4\underline{i} - 10\underline{j} + \underline{k}$ m/s

we have $|\underline{v}| = \sqrt{16+100+1} = \sqrt{117}$ m/s $= 3\sqrt{13}$ m/s

(9) given $\underline{v}_A = 5\underline{i} + 2\underline{j}$ m/s and $\underline{v}_B = -4\underline{i} + 4\underline{j}$ m/s

then $|\underline{v}_A| = \sqrt{25+4} = \sqrt{29}$ m/s

and $|\underline{v}_B| = \sqrt{16+16} = \sqrt{32}$ m/s

So particle B has the greater speed.

(11) given $\underline{v} = b\,\underline{i} + (b+7)\,\underline{j}$ m/s and $|\underline{v}| = 17$ m/s

we have $|\underline{v}| = \sqrt{b^2 + (b+7)^2} = 17$

$\therefore \quad b^2 + b^2 + 14b + 49 = 289$

$\Rightarrow \quad 2b^2 + 14b - 240 = 0$

$\therefore \quad b^2 + 7b - 120 = 0$

$\therefore \quad (b+15)(b-8) = 0 \quad \Rightarrow b = -15$ or $b = 8$

The question did not ask for this, but this means

$$\underline{V} = -15\underline{i} - 8\underline{j} \quad \& \quad \underline{V} = 8\underline{i} + 15\underline{j}$$

(13) given $\underline{r} = \overrightarrow{OP} = 5\underline{i} + 4\underline{j}$

Now, Final position vector = initial position vector + velocity vector over a given time $\circledast$

Note that "velocity vector over a given time" is in vector terms the same idea as $s = v.t$ in distance - speed terms.

$$\therefore \quad \underline{r}(t) = 5\underline{i} + 4\underline{j} + t(2\underline{i} - \underline{j})$$

$$= (5+2t)\underline{i} + (4-t)\underline{j} \quad m$$

When $t = 3$: $\underline{r}(3) = 11\underline{i} + \underline{j}$ m

When $t = 5$: $\underline{r}(5) = 15\underline{i} - \underline{j}$ m

(15) given $\underline{V} = 5\underline{i} - 12\underline{j}$ m/s

$$\therefore \quad |\underline{V}| = \sqrt{25 + 144} = 13 \text{ m/s}$$

By $\circledast$ of (13) : $\underline{r}_F(t) = r_0(t) + \underline{V}.t$, where $\underline{r}_F$ is the final position vector & $\underline{r}_0$ is the initial position vector.

So $\underline{r}_F(3) = \underline{i} + 6\underline{j} + 3 \cdot (5\underline{i} - 12\underline{j}) = 16\underline{i} - 30\underline{j}$ m

and $|\underline{r}_F| = \sqrt{256 + 900} = \sqrt{1156} = 34$ m.

given

(17) $\underline{r}_0 = a\underline{i} + b\underline{j} + c\underline{k}$ m

and $\underline{r}_F = 7\underline{i} + \underline{j} + 11\underline{k}$ m.

Since $\underline{r}_F = \underline{r}_0 + \underline{V}.t$ we have

$$7\underline{i} + \underline{j} + 11\underline{k} = a\underline{i} + b\underline{j} + c\underline{k} + 2.(3\underline{i} + \underline{j} + 4\underline{k})$$

$$= (6+a)\underline{i} + (2+b)\underline{j} + (8+c)\underline{k}$$

$\therefore$ compa left & Right sides : $7 = 6 + a \Rightarrow a = 1$

$1 = 2 + b \Rightarrow b = -1$

$11 = 8 + c \Rightarrow c = 3$

So $\underline{r}_0 = \underline{i} - \underline{j} + 3\underline{k}$ m

At $t = 3$: $\underline{r}_F = \underline{i} - \underline{j} + 3\underline{k} + 3.(3\underline{i} + \underline{j} + 4\underline{k})$

$$= 10\underline{i} + 2\underline{j} + 15\underline{k} \text{ m}$$

$$\therefore |\underline{r}_F| = \sqrt{100 + 4 + 225} = \sqrt{329} \text{ m}$$

(19) given $\underline{r}_A = 2\underline{i} + 3\underline{j} - 4\underline{k} + t_A(-\underline{i} + 3\underline{j} + 5\underline{k})$

& $\underline{r}_B = 8\underline{i} + 6\underline{k} + t_B.\underline{V}$

When $t = 4$: $\underline{r}_A = 2\underline{i} + 3\underline{j} - 4\underline{k} + 4(-\underline{i} + 3\underline{j} + 5\underline{k})$

$$= -2\underline{i} + 15\underline{j} + 16\underline{k}$$

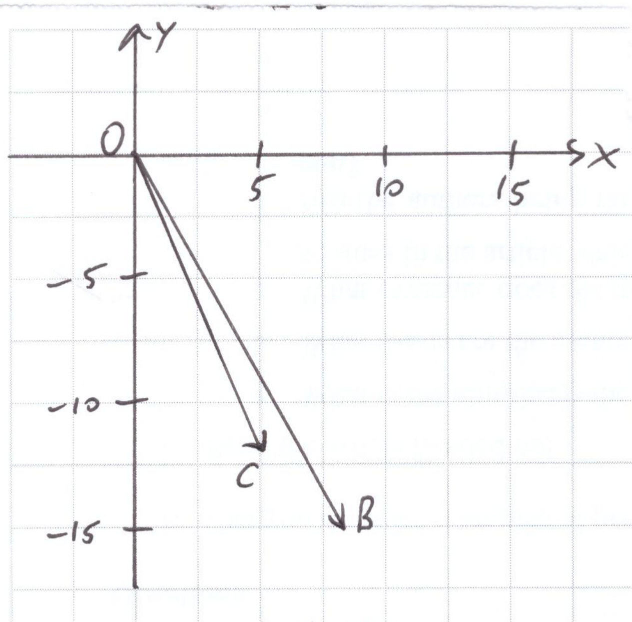
So

$\underline{r}_B = 8\underline{i} + 6\underline{k} + 5\underline{V}_B = -2\underline{i} + 15\underline{j} + 16\underline{k}$

$\Rightarrow \underline{V}_B = \frac{1}{5}(-10\underline{i} + 15\underline{j} + 10\underline{k}) = -2\underline{i} + 3\underline{j} + 2\underline{k}$

Ex 2B, P23 (even N^os^)

(2)



(a) $|\vec{OB}| = \sqrt{8^2 + (-15)^2} = 17$ m

(b) $|\vec{OC}| = \sqrt{5^2 + (-12)^2} = 13$ m

(c) $\vec{BC} = -\vec{OB} + \vec{OC} = (5\underline{i} - 12\underline{j}) - (8\underline{i} - 15\underline{j})$

$= -3\underline{i} + 3\underline{j}$

(d) $|\vec{BC}| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 4.24$ m

(4) $\underline{v} = (6\underline{i} - 8\underline{j})$ m/s. Speed $v = |\underline{v}|$

$= \sqrt{6^2 + (-8)^2}$

$= 10$ m/s

(6) $\underline{v} = (-4i + j)$ m/s.

Speed $v = |\underline{v}| = \sqrt{(-4)^2 + 1^2} = \sqrt{17} = 4.12$ m/s

(8) $\underline{v} = (3\underline{i} - \underline{j} - 7\underline{k})$ m/s

Speed $v = |\underline{v}| = \sqrt{3^2 + (-1)^2 + (-7)^2} = \sqrt{59}$

$= 7.68$ m/s

(9) $\underline{v}_A = (5\underline{i} + 2\underline{j})$ m/s ; $\underline{v}_B = (-4\underline{i} + 4\underline{j})$ m/s

$|\underline{v}_A| = \sqrt{5^2 + 2^2} = \sqrt{29}$ m/s

$|\underline{v}_B| = \sqrt{(-4)^2 + 4^2} = \sqrt{32}$ m/s

So $\underline{v}_B$ has the greater speed.

(10) $\underline{v} = (2\underline{i} + a\underline{j})$ m/s & $|\underline{v}| = 5.2$ m/s

So $|\underline{v}| = \sqrt{4 + a^2} = 5.2$

$\Rightarrow 4 + a^2 = 27.04$, $\therefore a^2 = 23.04$

$\Rightarrow a = \pm 4.8$

(12) $\underline{r}_0 = (5\underline{i} + 3\underline{j})$ m ; $\underline{v} = (2\underline{i} + 4\underline{j})$ m/s.

<u>Note</u>: Displacement from $\underline{r}_0$ = Starting point + (velocity) * t

i.e.

$$\underline{r} = \underline{r}_0 + \underline{v} \cdot t$$

So here $\underline{r} = (5\underline{i} + 3\underline{j}) + (2\underline{i} + 4\underline{j})t$

(a) after 1 sec: $\underline{r} = (5\underline{i} + 3\underline{j} + 2\underline{i} + 4\underline{j})$ m

$= (7\underline{i} + 7\underline{j})$ m

(b) after 2 sec: $\underline{r} = (5i + 3j + 4\underline{i} + 8\underline{j})$ m

$= (9\underline{i} + 11\underline{j})$ m

(14) Displacement = initial displacement + (vel)(time)

i.e. $\underline{r} = \underline{r}_0 + \underline{v}.t$

So at $t = 3$:

$10\underline{i} - \underline{j} = 7\underline{i} + 5\underline{j} + 3(a\underline{i} + b\underline{j})$

$\Rightarrow \quad 10\underline{i} - \underline{j} = (7 + 3a)\underline{i} + (5 + 3b)\underline{j}$

$\therefore \quad 10 = 7 + 3a \Rightarrow a = 1$

& $\quad -1 = 5 + 3b \Rightarrow b = -2$

(16) $\underline{r} = \underline{r}_0 + \underline{v}.t$

(a) $\underline{r} = 4\underline{i} + 3\underline{j} + 9\underline{k} + (3\underline{i} - 2\underline{j} - 5\underline{k}).t$

(b) at $t = 5$: $\underline{r} = 4\underline{i} + 3\underline{j} + 9\underline{k} + 15\underline{i} - 10\underline{j} - 25\underline{k}$

$= 19\underline{i} - 7\underline{j} - 16\underline{k}$

$|\underline{r}| = \sqrt{19^2 + (-7)^2 + (-16)^2} = \sqrt{666}$

$= 3\sqrt{74}$ m

(18) $\underline{r}_0 = (7\underline{i} - 6\underline{j} + 3\underline{k})$ m

after $t = 2$: $\underline{r}_1 = 7i - 6j + 3\underline{k} + 2(4\underline{i} - 6\underline{k})$

$= (15\underline{i} - 6\underline{j} - 9\underline{k})$ m

$\underline{r}_2 = \underline{r}_1 + v.t$

$\Rightarrow \underline{r}_2 = (15\underline{i} - 6\underline{j} - 9\underline{k}) + 3(a\underline{i} + b\underline{j} + c\underline{k})$

$= 0\underline{i} + 0\underline{j} + 0\underline{k}$ since $\underline{r}_2$ is at the origin.

So $0 = 15 + 3a \Rightarrow a = -5$

& $0 = -6 + 3b \Rightarrow b = 2$

& $0 = -9 + 3c \Rightarrow c = 3$